THE EINSTEIN THEORY.

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When your Programme Committee, through the President, asked me to read a paper on the subject of Relativity and the Gravitation Theory at the General Meeting of the Society, I assumed that it was in the thought that one who had occupied himself mainly with the study of concrete physical phenomena might be able to contribute something towards a definite physical conception of the new theory.

I shall not take more time to go into the question as to how the theory of relativity was developed than merely to say that a number of physical phenomena are known which appear to be in contradiction to the system of mechanics founded on Newton's laws of motion. Now Newton's laws are based upon the fundamental concepts of space, time and matter. The space of Newton is the space of Euclid—the space of our ordinary experience. The time of Newton is the time that we ordinarily think of—a conception wholly independent of our space conception. And matter for Newton is the matter that is perceived by our senses.

Equally fundamental in Newton's mechanics to the three concepts of space, time and matter is that of force—the cause of every change in motion. That the idea of force is as fundamental a notion to us as that of matter there is little doubt; they are both revealed to us by our senses; our muscular sense gives us very directly a realization of force. When, however, a system of mechanics is built up with force as one of the four fundamental concepts a certain indeterminateness arises. I need mention only the controversy that still goes on as to the exact interpretation of centrifugal force, and other forces that we have to consider that are certainly not the cause but the result of motion. And when we extend our system of mechanics so as to cover all physical phenomena forces of other kinds must be postulated—electric, magnetic, molecular, chemical forces—forces of which we have no direct sense, but which nevertheless must be regarded as having a real existence.

In an attempt to clear away the indeterminateness involved in the conception of force as fundamental, and the complexity inherent in a multiplicity of forces, Hertz developed a system of mechanics in which the idea of force as one of the fundamental concepts was banished. In this system of mechanics all forces are the result of constraints arising from concealed or cyclic motions. If we should experiment with a rapidly spinning wheel enclosed in a box, not knowing what there was in the box, we should come to the conclusion that the box was in a field of force quite different from a simple gravitational field; or in other words the potential energy of the box would appear to be different from its potential energy with the wheel at rest. But knowing of the wheel in rotation, what would appear as potential energy arising from an external field would really be kinetic energy of cyclic motion. So Hertz attempted to interpret every force acting on a system as arising from cyclic motions, with a single law governing the motion of the system—the law of the straightest path. There is a close relation between Hertz's system of mechanics and Einstein's theory of gravitation to which we shall return later.

Let us now go back to the Newtonian view and regard force as a fundamental concept. The force that we are most familiar with is the force of gravity. Newton showed that not only the motion of bodies falling to the earth, but the motion of the planets about the sun could be accounted for by assuming that every particle of matter in the universe attracts every other particle with a force proportional to the product of the masses and inversely proportional to the square of the distance between them. This was, of course, no explanation of the force of gravity, and the idea that matter could act upon matter at a distance was distasteful to Newton himself, as it has been almost universally ever since. However, up to about the middle of the nineteenth century it was considered a sufficient goal to attain in a variety of physical phenomena to account for them by means of forces acting at a distance between elements of the system. Particularly in the fields of electricity and magnetism this goal seemed near attainment. There was, however, a very real difficulty. In the attempt to account, on this principle, for the forces between circuits carrying electric currents, not one, but an infinite number of laws of force between the elements of the circuits was found to answer. Experiment could not decide which was the law of force because experiments could be made only with complete circuits. An end was soon put to the controversy which raged over this question by the publication of Maxwell's Theory of Electricity and Magnetism. In this theory action at a distance played no part. All the forces between electrically charged bodies, between magnetized bodies, the mutual forces between electric circuits and between magnets and electric circuits were ascribed to a system of pressures and tensions in a universal medium which pervaded all bodies and extended throughout all space. And this medium was the same as that which had been previously postulated as the vehicle for the waves of light. The goal in the dynamical explanation of physical phenomena now changed to the attempt to account for them by direct action through a medium instead of by action at a distance. For electric and magnetic effects the idea of action at a distance became unnecessary, but for the commonest force of all—gravitation—it could not be dispensed with. Although the elementary law of gravitational attraction is remarkably similar to the elementary laws of electric and magnetic attractions and repulsions, there are sufficient differences between them to place the force of gravity in a different category from the other natural forces. Gravitational force is always attractive; electric and magnetic forces may be attractive or repulsive; gravitational force appears to be wholly independent of the medium through which it acts; electric and magnetic forces are enormously influenced by the medium. These differences led Maxwell to predict that attempts to account for gravitational force by a system of pressures and tensions in a medium, analogous to those used to account for electric and magnetic forces, would be doomed to failure.

An answer to this riddle of gravitation has been given by Einstein, and this answer has come through the general theory of relativity. The principle of relativity has arisen through repeated failures to detect any influence upon optical phenomena by experiments performed on the earth due to the motion of the earth about the sun. In the same way, the two principles of physics that have kept their validity—the law of the conservation of energy and the second law of thermodynamics—grew out of the failure to find any violations of them. As is the case with these two laws, the principle of relativity may be stated in a number of alternative ways. From the gravitational point of view Jeans has stated this principle "A planet cannot describe a perfect ellipse about the sun as focus," and this statement expresses very distinctly the failure of the Newtonian mechanics to account for all known physical phenomena.

Now instead of trying to modify Newton's laws of motion, Einstein goes back of them and uses views of space and time which are different from those upon which the Newtonian mechanics is founded. For the purpose of describing natural phenomena the Euclidean space has almost universally been considered sufficient. Whether or not Euclidean space represents anything which has a real existence has been a doubtful question among mathematicians from the earliest times. Other systems of geometry have been developed, following closely the plan of Euclid, keeping some of his axioms and rejecting others, and the consequences examined. Riemann, however, in his essay on the "Hypotheses which are the Foundation of Geometry" introduced a new system of geometry, and the development of Riemann's geometry supplied the altered conception of space and time necessary for the Einstein theory.

The Riemann geometry bears a relation to Euclidean geometry somewhat analogous to the relation of direct action to action at a distance in physics. According to Riemann, space is a three-dimensional continuum, by which is meant that a point in space may be represented continuously by three independent quantities, the coördinates of the point. Riemann considered the more general problem of a continuum in which n independent coördinates are required to specify a point, thus developing an n-dimensional geometry. In order to define the metrical properties of space Riemann assumed that the square of the distance between two infinitely near points is a quadratic differential form of the relative coöordinates of the points, with coefficients not constant, but functions of the coördinates. In Euclidean space it is always possible to choose coördinates—the usual rectangular coördinates—such that the square of the distance between any two points shall be expressed as the sum of the squares of the relative coördinates of the two points. In the generalized space of Riemann this cannot be done. An analogy will make this distinction clear. A plane in three-dimensional space may be regarded as Euclidean space of two dimensions, for by choosing any rectangular coördinates in it it is possible to express the square of the distance between two points as the sum of the squares of the relative coördinates of the points. A curved surface in three dimensions, however, is non-Euclidean space of two dimensions, for the distance between two points on the surface measured along the surface cannot be expressed in the same way as on a plane. The geometry of curved surfaces in three-dimensional space was developed by Gauss, and Riemann's geometry is an extension of the Gaussian methods to surfaces of a greater number of dimensions. In this way the conception of curvature of space arose, as a perfectly logical development of the easily conceived curvature of a surface. Space of zero curvature is Euclidean space; if the curvature is different from zero, whether constant or varying from point to point, space is non-Euclidean. Measurements on a two-dimensional surface will tell whether the surface is plane or curved—that is, whether it is Euclidean space or not. For by measuring the circumference of a circle drawn on the surface with a known radius, if the circumference is 2π times the radius, the surface is plane. If the surface is curved the result will in general be different. So it might be thought that measurements in our actual three-dimensional space would tell whether our space is Euclidean or not. In fact, Gauss did attempt to test this question by carefully measuring the angles between three distant points, but needless to say he found no departure from Euclidean space.

We must now consider the question of time. Until Lorentz introduced what he called the "local time" in his theory of electrical and optical phenomena in moving bodies, and thus laid the foundation for the theory of relativity, time and space were regarded as wholly independent concepts, at least for the purpose of describing physical phenomena. Our knowledge of the physical universe we obtain by experience, and it is certainly true that no one ever determined a position in space except at a definite time, nor noted a time except at a definite position in space. It was Minkowski who first clearly stated that to define an event four generalized coordinates are needed—three to define its place in space and one to define its time. The universe thus becomes, in Riemann's sense, a four-dimensional continuum.

The expression for the square of the line element in this generalized space is a quadratic differential form with ten terms. The coefficients in this expression determine the departure of this generalized space from Euclidean space. In order to satisfy the condition for the complete relativity of physical phenomena it is necessary that this line element shall have the same value in whatever system of coördinates it is measured. Now in Einstein's theory these coefficients have more than a purely geometrical significance. They have a dynamical meaning in that they determine the gravitational field. Or to put it in another way, the curvature of space is determined by the presence of matter. At a great distance from all matter this four-dimensional space is Euclidean. The presence of matter gives to space its curvature. We can now see how, for gravitational forces, the goal of the Hertzian mechanics is attained, although in a wholly different way from that contemplated by Hertz. Gravitational forces, according to Einstein, do not exist. Hertz's law of the straightest path has universal validity in this system, but the straightest path may appear to be a curved path because it must be drawn in space which is curved. In the two-dimensional analogue the straightest path between two points on a curved surface is not the straight line connecting the two points, for that line would take us out of our space. The straightest path is the geodesic drawn on the surface between the two points. And so light rays passing close to the sun are not attracted by the sun, but the space through which they pass being curved under the influence of the mass of the sun, the rays follow a curved path in reaching the earth.

Now any theory of this kind to be at all complete cannot stop with explaining away gravitational forces. Electric and magnetic forces, which we have seen differ in their nature from gravitational forces, must also be considered. I can only mention a remarkable extension by Weyl of the Einstein theory, which is really a logical extension of the Riemann geometry. In Riemann's geometry the scale of measurement is fixed; a line element at one place can be compared directly with a line element at a distance. But in a system of geometry to remain true to the idea of direct action as opposed to action at a distance this assumption appears unwarranted. And so Weyl assumes that the scale of measurement varies from point to point in the four-dimensional universe. This hypothesis results in another differential form which characterizes the metrical properties of space—this time a linear differential form—and Weyl shows how the electric and magnetic state of space can be interpreted in terms of the coefficients which enter into this expression.

All that I have attempted to do in the foregoing is to show what kind of a theory the Einstein theory is; how radically it differs in principle from what we are accustomed to ask for in a physical theory. This theory opens up to the study of natural phenomena a new universe, a universe in which geometry and physics cannot be regarded as independent sciences. This universe is a four-dimensional metrical manifold; physical phenomena are determined by the metrical properties of this universe. There is no reason that I can see why this generalized space of the Einstein theory should not be named "the ether." But giving it a name does not help in understanding its properties, and it is a wholly different ether from that to which we have grown accustomed.

It is interesting to note that the possibility of space having a dynamical property was suggested by Riemann, although it was left for Einstein to develop the consequences of such a conception. In fact, Riemann went farther, and suggested the possibility of space being a discrete manifold instead of a continuum, and this suggestion is of particular interest at the present time in view of the growing importance of the quantum theory which is founded on the idea of discreteness somewhere as opposed to continuity.

The difficulties in understanding the Einstein theory are not so much mathematical difficulties; they arise from the vain attempt to picture to our minds the kind of space required by the theory. We instinctively try to form a model of some mechanism which will give us a representation of natural phenomena; but according to Einstein the materials we have hitherto used to form such models—our conceptions of space, time and matter—are inadequate. If his theory is to stand we must make a new universe in our minds, a universe in which space and time have an existence only when considered in their relation to matter.